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ABSTRACT

A Chebyshev-Fourier discretization with shock fitting is used to solve the unsteady Euler equations. The method is applied to shock interactions with plane waves and with a simple model of homogeneous isotropic turbulence. The plane wave solutions are compared to linear theory.

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1. INTRODUCTION

The effects of a shock wave upon a turbulent boundary layer have been studied extensively by both experimental and computational means. A persistent experimental result is that the turbulence levels downstream of the shock are higher, by at least a factor of 2, than the upstream levels [1]. The computational results, however, have consistently failed to exhibit this dramatic turbulence amplification [2]. This failure is almost surely due to inadequacies in the turbulence models used in the computations. Current turbulence models have achieved their greatest success in low-speed flows and attached high-speed flows which agree well with experiment even up to Mach 20 [3]. Evidently, some essential physics of separated compressible flow is missing from the models. Their improvement must await a better understanding of the interaction between shock waves and turbulence.

Two theoretical tools are now available for the study of the inviscid aspects of this interaction. The basically linear effects may be examined analytically in a way developed by Ribner [4] and others, and used by Anyiwo and Bushnell [5] in their assessment of turbulence amplification in the shock-boundary layer interaction. Numerical computations based on the nonlinear, two-dimensional Euler equations are more recent. Pao and Salas [6] used this approach to study the shock-vortex interaction. So did Zang, Hussaini and Bushnell [7] in their examination of the interaction of plane waves with shocks. Both sets of numerical computations used second-order finite differences for the spatial discretization. In this paper we will describe a pseudospectral spatial discretization.

2. NUMERICAL METHOD

The model problem which is used to study the turbulence amplification and generation mechanisms is illustrated in Figure 1. At time $t = 0$ an infinite, normal shock at $x = 0$ separates a rapidly moving, uniform fluid on the left from the fluid on the right which is in a quiescent state except for some specified fluctuation. The initial conditions are chosen so that in the absence of any fluctuation the shock moves uniformly in the positive x-direction with a Mach number

(relative to the fluid on the right) denoted by M_s. In the presence of fluctuations the shock front will develop ripples. The structure of the shock is described by the

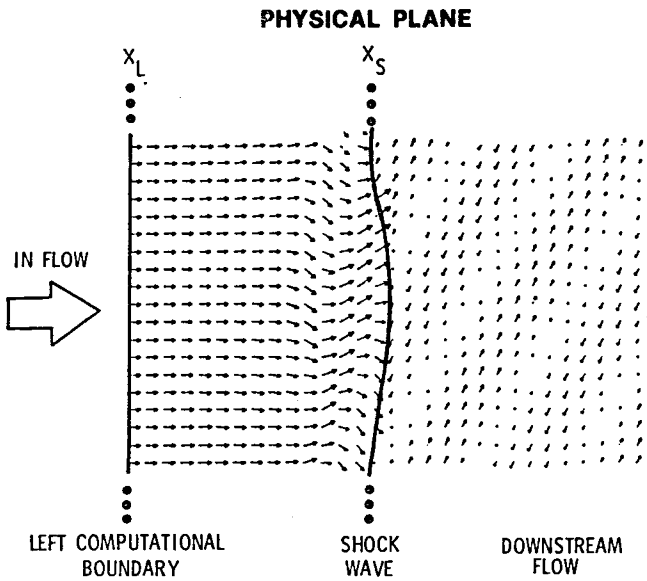

Figure 1: Model problem in the physical domain.

function $x_s(y,t)$. The numerical calculations are used to determine the state of the fluid in the region between the shock front and some suitable left boundary, $x_L(t)$, and also to determine the motion and shape of the shock front itself.

In the applications given below the physical problem is periodic in y with period y_L. The change of variables

$$\begin{aligned} X &= \frac{x - x_L(t)}{x_s(y,t) - x_L(t)} \\ Y &= y/y_L \\ T &= t \end{aligned} \tag{1}$$

produces the computational domain

$$\begin{aligned} 0 &\leq X \leq 1 \\ 0 &\leq Y \leq 1 \\ T &\geq 0. \end{aligned} \tag{2}$$

In terms of the computational coordinates the two-dimensional Euler equations are

$$Q_T + BQ_X + CQ_Y = 0, \tag{3}$$

where $Q = (P,u,v,S)$,

$$B = \begin{bmatrix} U & \gamma X_x & \gamma X_y & 0 \\ \frac{c^2}{\gamma} X_x & U & 0 & 0 \\ \frac{c^2}{\gamma} X_y & 0 & U & 0 \\ 0 & 0 & 0 & U \end{bmatrix} \tag{4}$$

and

$$C = \begin{bmatrix} V & \gamma Y_x & \gamma Y_y & 0 \\ \frac{c^2}{\gamma} Y_x & V & 0 & 0 \\ \frac{c^2}{\gamma} Y_y & 0 & V & 0 \\ 0 & 0 & 0 & V \end{bmatrix}. \tag{5}$$

The contravariant velocity components are given by

$$U = X_t + uX_x + vX_y$$

and (6)

$$V = Y_t + uY_x + vY_y.$$

A subscript denotes partial differentiation with respect to

the indicated variable. P, c and S are the natural logarithm of pressure, the sound speed and the entropy (divided by the specific heat at constant volume), respectively, all normalized by reference conditions at downstream infinity; u and v are velocity components in the x and y directions, both scaled by the characteristic velocity defined by the square root of the pressure-density ratio at downstream infinity. The ratio of specific heats is denoted by γ; a value $\gamma = 1.4$ has been used for all the calculations in this paper.

Let n denote the time level and Δt the time increment. The time discretization of Eq. (3) is

$$\tilde{Q} = \left[1-\Delta t L^n\right]Q^n \tag{7}$$

$$Q^{n+1} = \frac{1}{2}\left[Q^n+\left(1-\Delta t\tilde{L}\right)\tilde{Q}\right], \tag{8}$$

where L denotes the spatial discretization of $B\partial_X + C\partial_Y$. The solution Q has the Chebyshev - Fourier series expansion

$$Q(X,Y,T) = \sum_{p=0}^{M} \sum_{q=-N/2}^{N/2-1} Q_{pq}(T)\tau_p(\xi)e^{2\pi iqY}, \tag{9}$$

where $\xi = 2X-1$ and τ_m is the Chebyshev polynomial of degree m. The derivatives Q_X and Q_Y are approximated by

$$Q_X = 2\sum_{p=0}^{M} \sum_{q=-N/2}^{N/2-1} Q_{pq}^{(1,0)}(T)\tau_p(\xi)e^{2\pi iqY} \tag{10}$$

$$Q_Y = 2\pi\sum_{p=0}^{M} \sum_{q=-N/2}^{N/2-1} Q_{pq}^{(0,1)}(T)\tau_p(\xi)e^{2\pi iqY} \tag{11}$$

where

$$Q_{pq}^{(1,0)} = \frac{2}{c_p} \sum_{\substack{m=p+1 \\ p+m \text{ odd}}}^{M} m\, Q_{mq} \tag{12}$$

with

$$c_p = \begin{cases} 2 & p = 0 \\ 1 & p > 1 \end{cases} \tag{13}$$

and

$$Q_{pq}^{(0,1)} = iq\, Q_{pq}. \tag{14}$$

The most critical part of the calculation is the treatment of the shock front. The shock-fitting approach used here is desirable because it avoids the severe post-shock oscillations that plague shock-capturing methods. The time derivative of the Rankine-Hugoniot relations provides an

equation for the shock acceleration. This equation is integrated to update the shock position. The jump conditions themselves are then used at $x_s(y,t)$ to determine the variables on the left (high perssure) side of the shock from the specified downstream flow (see [8] for details). For $M_s > 2.08$, the left boundary at $X_L(t)$ is a supersonic inflow boundary. Lower Mach numbers require more sophisticated boundary conditions as discussed in [8]. Spectral methods for compressible flows typically require some sort of filtering.

3. INTERACTION OF PLANE WAVES WITH A SHOCK

The non-linear interaction of plane waves with shocks was examined at length in [7]. The numerical method used there was similar to the one described above but employed second-order finite differences in place of the present pseudospectral discretization. Detailed comparisons were made in [7] with the predictions of linear theory [4]. The linear results turned out to be surprisingly robust, remaining valid at very low (but still supersonic) Mach numbers and at very high incident wave amplitudes. The only substantial disagreement occurred for incident waves whose wave fronts were nearly perpendicular to the shock front. This type of shock-turbulence interaction is a useful application of the pseudospectral technique. The method can be calibrated in the regions for which linear theory has been shown to be valid. The questionable regions can then be gainfully investigated by means of this alternative method.

We concentrate on Mach 3 normal shocks interacting with incident vorticity waves. The perturbation velocities ahead of the shock are taken to be

$$(u',v') = A'(-k_{1,y},k_{1,x})(c_1/p_1 k_1)\cos(\underline{k}_1 \cdot \underline{x}),$$

where the wavevector $\underline{k}_1 = (k_{1,x},k_{1,y})$ and A' is the amplitude. In terms of the incidence angle θ_1, $\underline{k}_1 = (k_1 \cos\theta_1, k_1 \sin\theta_1)$. The transmitted vorticity behind the shock can be expressed in the same manner with all subscripts 1 changed to 2. The vorticity transmission coefficient is defined to be the ratio A_2'/A_1'. At Mach 3 the ratio of the rotational part of the perturbation velocities is 0.158 times the transmission coefficient; if these perturbation velocities are scaled by the respective mean velocities (in the frame in which the shock is at rest), then this ratio is 0.611 times the transmission coefficient.

A typical numerical simulation is shown in Figure 2. At time $t = 0$ all the fluid in the computational domain, which then extended from $x_L = -0.5$ to $x_s = 0.0$, was in uniform

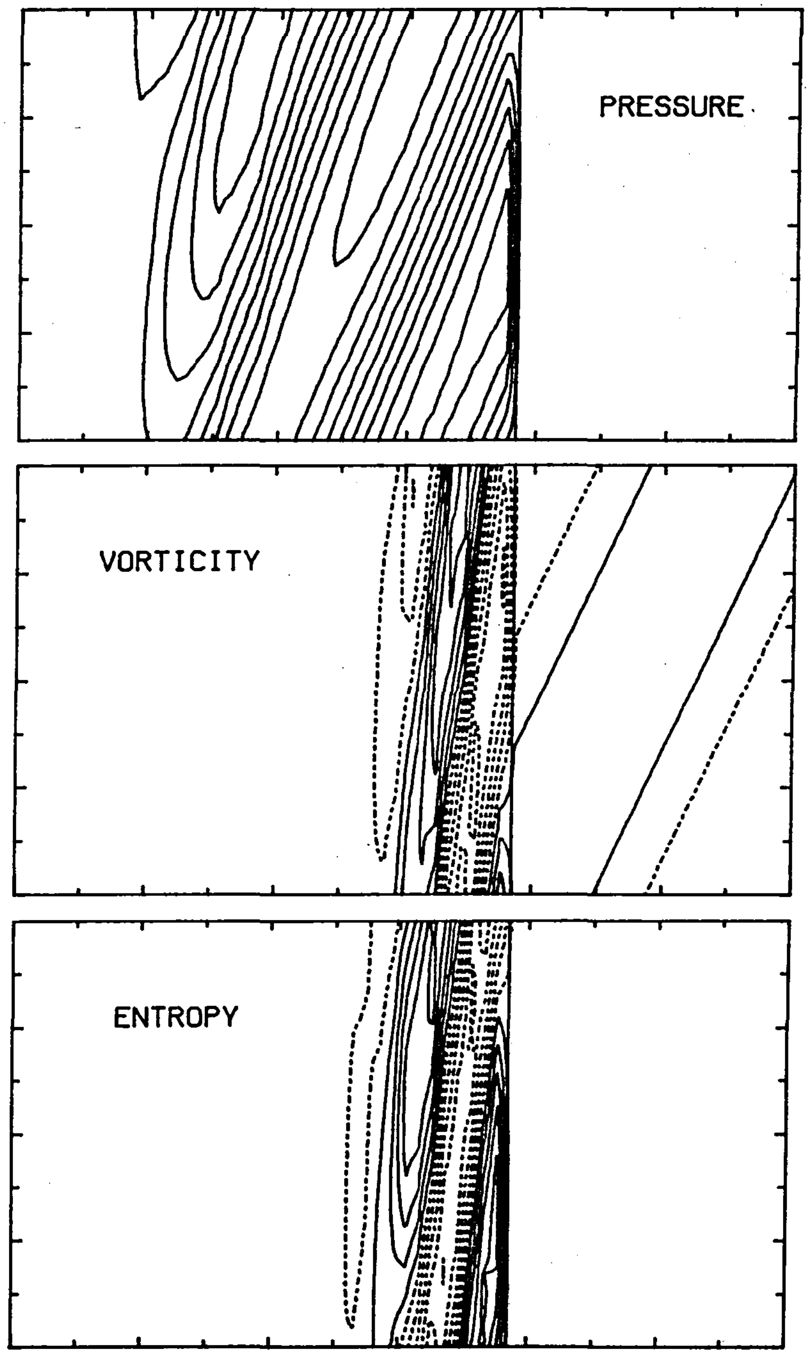

Figure 2: Responses at time $t = 0.80$ to a 0.1% vorticity wave striking a Mach 3 shock at a 30^0 angle of incidence. The shock front is denoted by the solid line connecting the top and bottom boundaries. Dashed lines indicate negative contour levels.

flow. The flow ahead of the shock was given by Eq. (15) with an amplitude $A' = 0.001$. In order to reduce the transients that would result if the fluid were to encounter this wave in a sudden fashion, the amplitude was turned on smoothly from $A' = 0.000$ to $A' = 0.001$ during the passage of the shock over the first half-wavelength of the incident wave. The interaction of the shock with the incident vorticity wave has produced a transmitted vorticity wave as well as generated acoustic and entropy waves. The acoustic wave clearly has a larger inclination angle and a faster propagation speed than the other two waves.

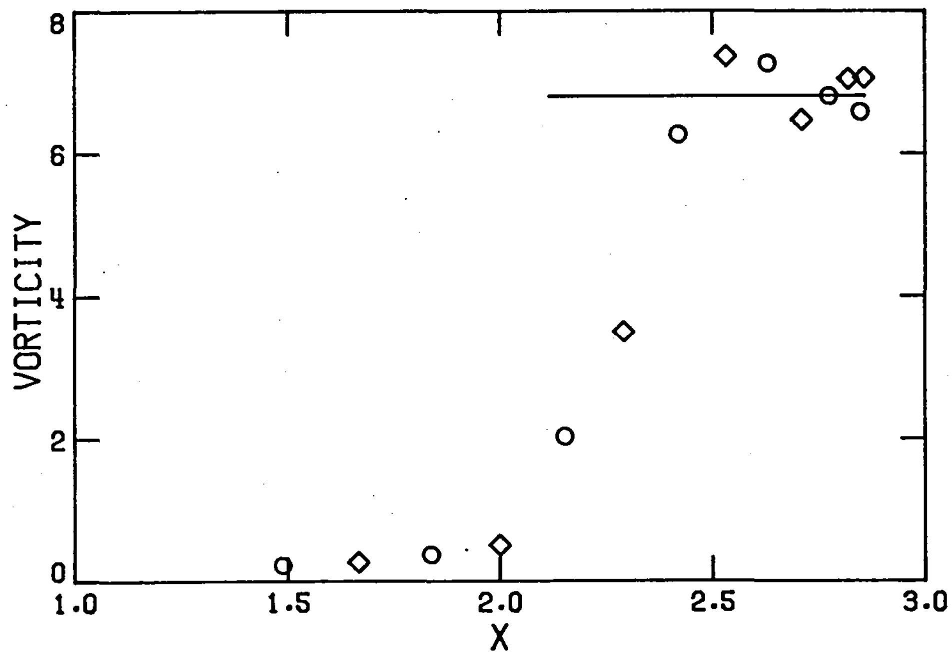

Figure 3: Post-shock dependence at time $t = 0.80$ of the vorticity response to a vorticity wave incident at 30^0 to a Mach 3 shock. The solid line is the linear theory prediction. Computed responses are given for 0.1% (circles) and 50% (diamonds) waves.

Figure 3 indicates how we measure the transmitted vorticity. The computed velocities are differentiated pseudospectrally and combined to produce the response vorticity at each point on the computational grid. At each fixed value of X we perform a Fourier analysis in Y of the vorticity. The wavenumber of interest is $q = 1$ (see Eq. (9)). This amplitude is then scaled as indicated above to yield A_2' and thence the ratio A_2'/A_1' which is plotted in Figure 3 as a function of a typical value of the physical coordinate x corresponding to X. In the case shown in Figure 3 the incident wave does not reach full strength

until $t = 0.56$. A single value representing the transmission coefficient is obtained by averaging the x-dependent responses. The standard deviation of these values serves as an error estimate, indicated by the error bars in the following two figures.

The angular dependence of the vorticity transmission coefficient for Mach 3 is displayed in Figure 4. Corresponding results from the finite difference discretization used in [7] are shown for comparison. The amplitude dependence for 30^0 incident waves is shown in Figure 5.

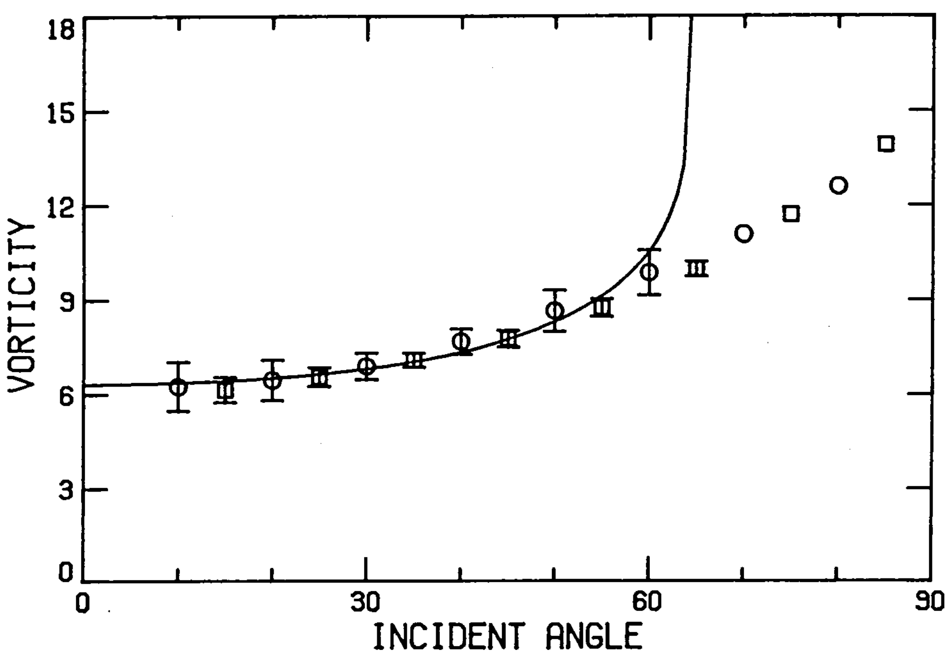

Figure 4: Incident angle dependence of the vorticity response to 0.1% vorticity waves incident upon a Mach 3 shock. Pseudospectral (circles) and finite difference (squares) results are given. The solid line is the linear theory prediction for sub-critical angles.

Linear theory predicts that the vorticity responses are peaked in the vicinity of $\theta_1 = 62^0$, the so-called critical angle. At larger angles the acoustic responses are evanescent. These predictions do not appear in the non-linear calculations. Tentative explanations for this failure of linear theory are provided in [7].

Below the critical angle, however, the linear theory is extremely robust. Figure 5 indicates that it remains valid for incident relative velocity fluctuations aproaching 50%.

(In the frame in which the shock is at rest, the ratio of the fluctuating velocity to the mean velocity is one-third of the amplitude A_1'.)

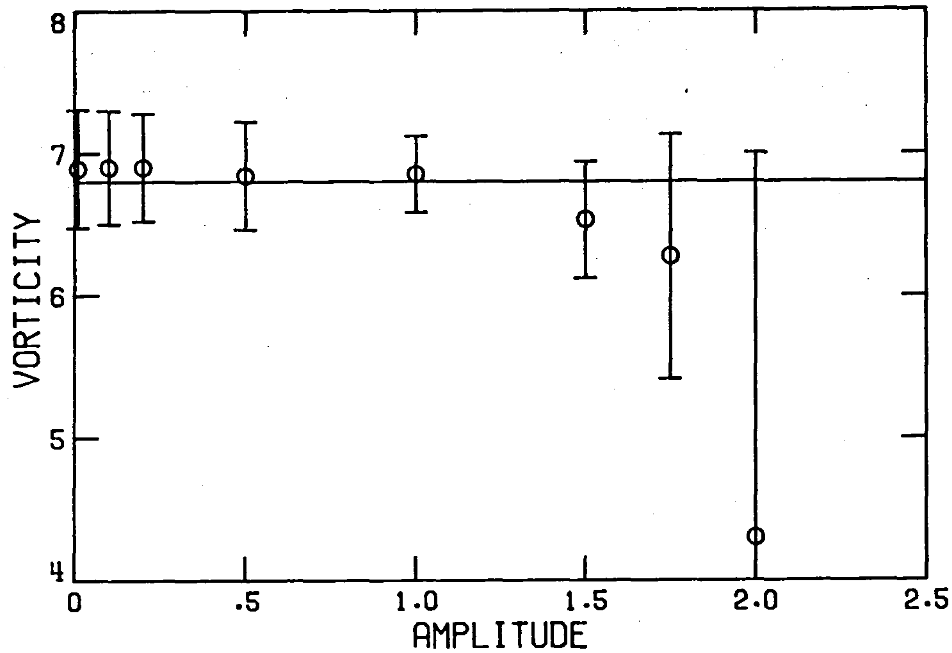

Figure 5 Amplitude dependence of the vorticity response to vorticity waves incident at 30^0 to a Mach 3 shock. The solid line is the linear theory prediction.

4. INTERACTION OF TURBULENCE WITH A SHOCK

Low intensity turbulence can be viewed as the superposition of plane shear waves just discussed. Figure 6 shows velocity vector plots of the interaction of a shock with a simple homogenous isotropic turbulence model in which the velocities are defined as sums of sinusoidal waves with random phases. The frame at $t = 0.38$ shows the undisturbed velocity field downstream of the shock and $t = 1.6$ shows the result after the shock has traveled approximately one wavelength of the longest wave component of the undisturbed turbulence. The development of the post-shock vortices seen in the figure coincides with the generation of rows of pressure and entropy spots behind the shock.

5. CONCLUSION

We have solved the two-dimensional unsteady non-linear Euler equations with a Chebyshev-Fourier pseudospectral

method. The linear theory of the interaction of a shock and plane shear waves has provided a test of the method. Details of the smoothing methods and questions of accuracy will appear in a later paper. But accuracy comparable to the finite difference calculations can be obtained with about half the number of Chebyshev modes in X and with only four modes in Y. The results indicate that it should be possible to compute more complicated shock-turbulence interactions.

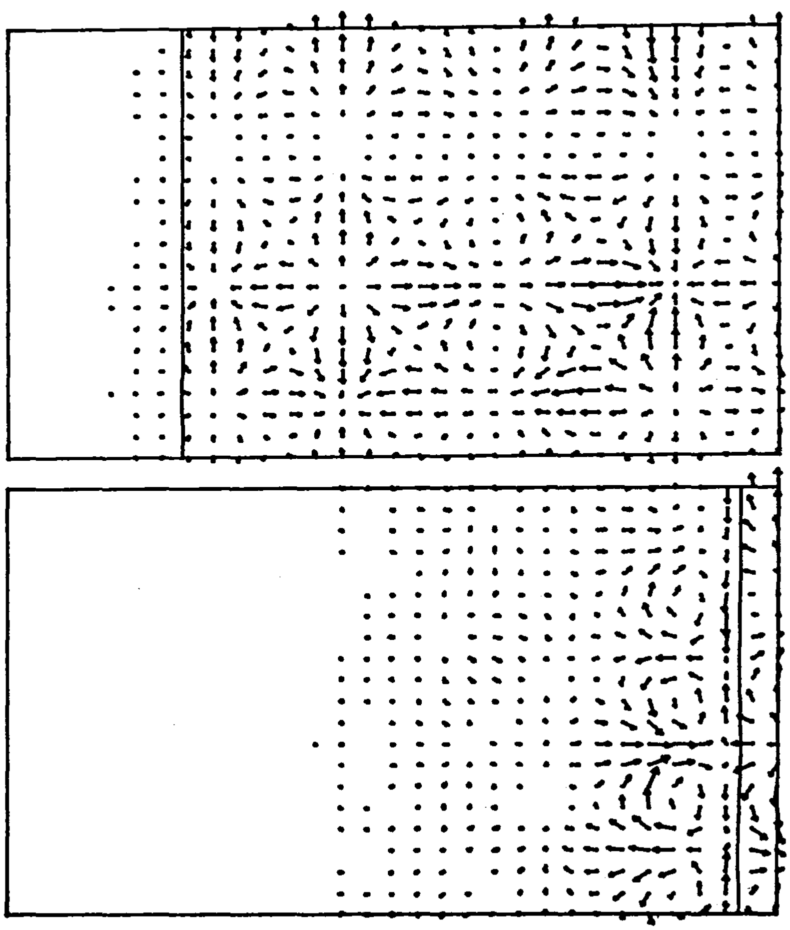

Figure 6: Velocity vector plots at t = 0.38 (top) and t = 1.6 (bottom) of the interaction of a Mach 3 shock with low intensity homogeneous isotropic turbulence.

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16. Abstract

A Chebyshev-Fourier discretization with shock fitting is used to solve the unsteady Euler equations. The method is applied to shock interactions with plane waves and with a simple model of homogeneous isotropic turbulence. The plane wave solutions are compared to linear theory.

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